

## **Theory Emerging Instability via Creative Perturbation – the Engine of Socio-Economic Progress**

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### **Abstract:**

*„I have the right to suppose that a thing is possible so long as its impossibility is not proved“.*

Leibniz, The Theodicy, 1710  
(Answer to II.Ob.Abridgment)

There are many misunderstandings in economics run from classics up to current time. Among other, they are concerning the axiologic evaluation of equilibrium/disequilibrium and stability/instability for appropriate socio/economic development. The majority of valuation is coming to ends that disequilibrium and instability having negative signs and so they are harmful for socio/economic development. Some of the reason of that depressed state of the subject in view of author is associated with inconsistent formulation of original mental models of one authentic economists and reading them with other one he/her has own mental model inconsistently formulated too. For demonstration of the trouble in the event of economic instability, the author uses examples of misunderstandings between Keynesians and neo-classicists, perhaps even monetarists and/or entirely in mainstream economics. Nevertheless, his main contemplation in this essay focused on axiological question of perceiving value of instability for socio-economic evolution: is instability negative or positive factor even another but what? Connected with that question his own conviction written in the heading of essay – the instability may be creative – consequently he purport that instability is decisive engine of socioeconomic progress. In other words in stable state can't occur any important qualitative change and positive socio-economic one by no means. The future successions of economy are the fruit of today planting. The author indicates that there are at least two virtual “technology” to investigate by the aid of CI the problem in question. The first and more conventional one is analysing character of formalized mental models by appropriated mathematical method and by simulation experimentation in virtual laboratories. Those models are the product of building it by method from up to down that is the product is wholly and directly the product of human subject. The second one is entirely new nonconventional “technology” based on method from bottom to up. In that case the human subject creates only virtual environment for evolution of creature by autopoiesis with minimisation of preliminary subjective imbeds to evolving entity. One of possible realization of that technology is for example methods and tools of MAS. The author in this essay uses the first referred simpler approach because it is more suitable for understanding in sight. For these purposes he use a few well-known mental models present in economic literature.

**Keywords:** bailout, bankruptcy, catastrophe theory, common law, creative perturbation, derivatives, disclosure, financial crisis, chaos, instability, leverage, monopoly, priority, security interest, socio-economic progress, systemic risk, transparency.

**JEL classification:** C7, D2, D8, E2, E4, E6, G2, G3, K1, K2, K4, N2

### **1. Introduction**

*Without time orientation to the future  
there is nil progress in economy.*

There are several approaches to instability derived from formula of stability in contemporary science. Among others we appoint *Lyapunov stability* (asymptotic stability of dynamical systems), and that which is principally important for our purpose the regime of *Structural stability* of economy. The last belongs to the category of global stability of complex evolutionary systems. It is a little difficult verbally describe the qualitative changes in structural stable/unstable evolutionary systems; however in mathematic approach (in topology) the description by changing shapes of lines it is understandably. Moreover such changing qualitative spectacles can be analyzed by exact mathematical procedures. Verbally mathematics say: when exactly done very small value of perturbation brings of the existing shape of the system to pass their topological identity, the result is structural change of stability, so such system is structurally unstable in done magnitude of perturbation. In the situation where magnitude of perturbation is smaller than meant done value the system remain structurally stable forever. Maybe it is for somebody surprise that merely simple mathematical formula can generate very complex behavior in iterative process. A. Agliari is the author of very instructive drafts, we using them below. In fig. 1 left, the draft show a cyclical attractor connected to a saddle cycle. The six attractive branches  $\omega_1$  of six saddles plucked away from closed invariant curve (CIC). The twin six attractive branches are coming from outer sphere of saddle cycle. The twelve repellent branches are winding around neighbours six local attractive CIC from both side. One can intuitively imagine that in their inner area there is some attractive periodic point. In fig. 2 is the draft showing situation at bifurcation: the merging of the outer branches of the invariant sets gives a *saddle connection*.

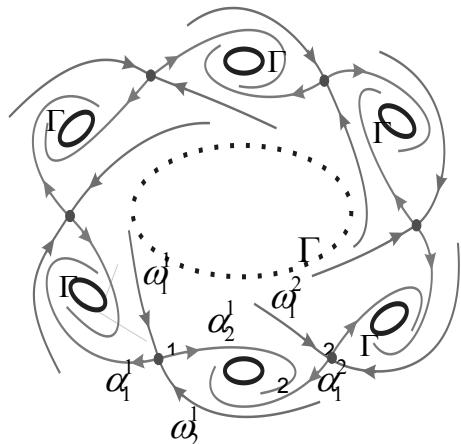


Fig. 1

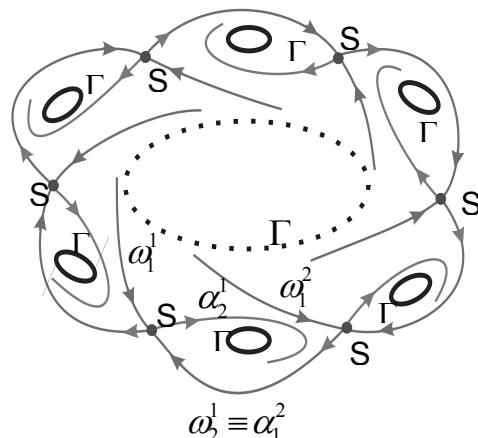


Fig. 2

The outer repellent branches of six saddles come about attractive one for neighbour addle in clockwise direction, that is branches becoming one is we see in bottom of fig.

2 ...  $\omega_2^1 \equiv \alpha_1^2$ . In fig. 3 is the draft of an attracting closed curve which appears, surrounding the cyclical attractor in their changed former shape to those one in fig. 2

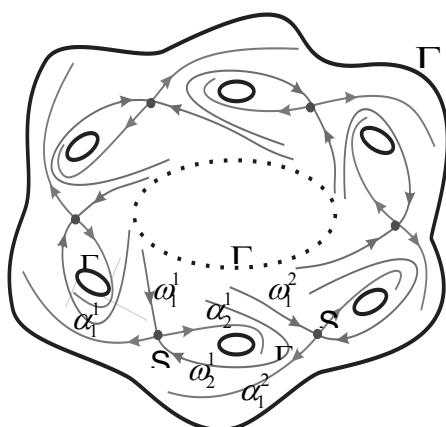


Fig. 3

Because we are using the term evolution<sup>1</sup> to descript process where emerging qualitative changes the phenomena of structural stability-unstability is very important for understanding such behaviour. In subsequent part we are demonstrating upper drafts using similar model from the area of financial market theory by simulation experiments in environment of iDMC.

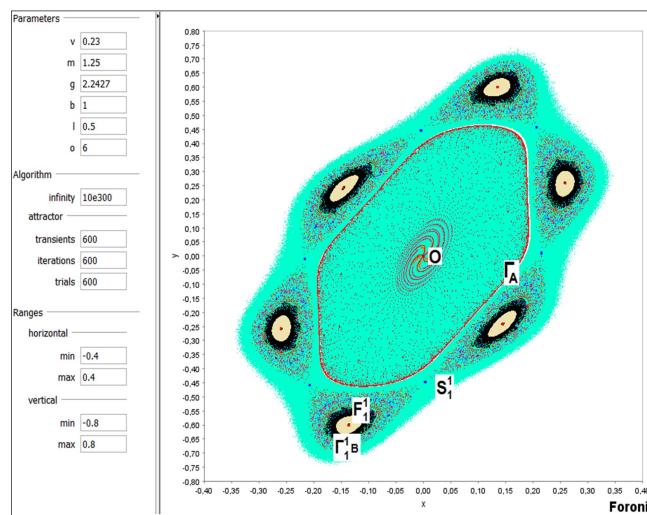


Fig. 4

To understand the change in future step in evolution it isn't enough to know what decision realised but it is important to know what motive based those decisions, that is what and why such expectation used by decision makers. In macroeconomics the subject of expectation is routinely the national income or DDP, that is completely abstract entity. Such abstraction fully dissembles the very source of that quantitative result of development in question, however. To be legible for customarily thinking economists we have to accept that mechanistic approach, that is the line of reasoning starting by J. M. Keynes, followed by R. Harrod, J. R. Hicks, J. Robinson and follow-up Keynesians and/or neo-Keynesians. Come across as reasonable to base the argumentation on famous Samuelson's multiplier-accelerator model (Samuelson, 1939) simply reformulated by F. H. Westerhoff (Westerhoff, 2006). Intended business cycle model of Samuelson introduced new insight on model of Keynes by using discrete time. He signed national income at time  $t$  as  $Y_t$ , which is written as the sum of three components, that is consumption  $C_t$ , induced private investment  $I_t$ , and governmental expenditure  $G_t$ . Therefore  $Y_t$  written by equation

$$Y_t = C_t + I_t + G_t \quad (1)$$

and in accord with the Keynesian multiplier analysis, the consumers use a constant fraction of their past income to buy items for consuming

$$C_t = a Y_{t-1} \quad (2)$$

where  $0 < a < 1$  is a parameter of conventional marginal propensity to consume. Induced private investment is proportional to changes in consumption and thus also to changes in national income

$$I_t = b(C_t - C_{t-1}) \quad (3)$$

<sup>1</sup> That term is another then term evolution used in contradictory procesess evolotion versus revolution.

and by substitution of  $Y$  from (2) beside  $C$  in (3) we obtain

$$I_t = ab(Y_{t-1} - Y_{t-2}). \quad (4)$$

which is very doubtfully assumption from the point of economic evolution (Samuelson is hanging on Keynes' "principle of fatal death in our future existence")? In fact, the time perspective of majority of people is oriented to the future in contrast to Keynes. His time perspective focused to the past and similarly so all economists of so-called *mainstream*. The strongest future time perspective is proper to entrepreneurs not to consumers. The instability arising from the changes of marginal propensity to consume has negative sign and lead to economic declination because the roots of marginal propensity to consume joint by what went before. The direction (arrow) of economic evolution based upon what has to happen in the future. Real positive instability is the product of entrepreneurs' challenging against uncertainty because they transforming the inventions to new process and products (horizontal diversification) and process and product innovations (vertical diversification). Their activities are results in inner positive perturbation that is the cause of directing economy to potential losing of former structural stability. Among very strong factors of positive external to entrepreneur's level perturbation, we are reckoning government investment to education, research and development. On the other hand, of bad direction in using government funds lead to economic decline. By the way in contemporary situations some of prognosis of economic development spread bad memetic infections in the economy. In some cases, they spread panic in financial market and after, consequently disaster in global economy as a whole. Such disaster isn't cyclical economic crisis, term that is misreads not only among economist but also in community generally. Actually that process in questions has character like one of the shapes of mathematical theory of catastrophe, which is nominally *cusp catastrophe* having typical *hysteretic loop*.

In Samuelson model the relation between the capital stock and output is fixed (obviously product of  $a$  by  $b$  must be  $ab > 0$ ). Because he set governmental expenditure constant

$$G_t = \bar{G} \quad (5)$$

national income rewritten to

$$Y_t = \bar{G} + a(1+b)Y_{t-1} - abY_{t-2} \quad (6)$$

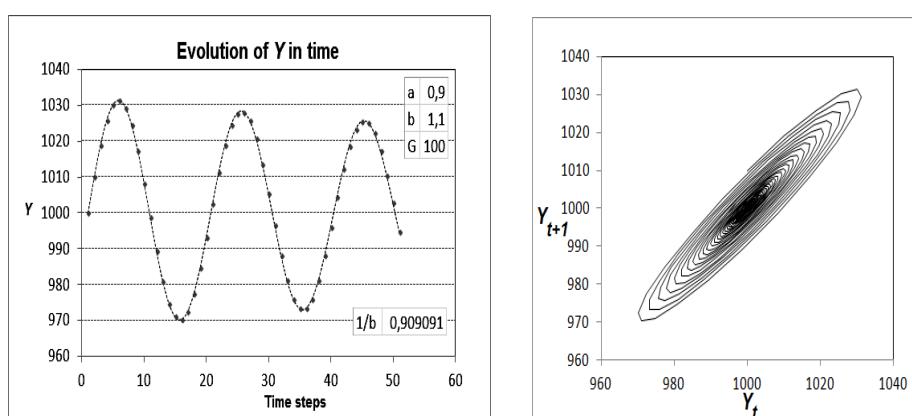


Figure 5

Samuelson's mental model, that is his virtual economy with his unrealistic assumptions actually show regular periodic oscillation. The economy is routing by damping oscillation to

equilibrium in a long period. For convenience of economists using Excel we show the process in spread sheet simulation as two snapshots in fig. 5 it is.

## 2. Introducing perturbation to Samuelson model by different expectation of the future

*Without destabilisation via creative perturbation of former stable equilibrium the progress is inconceivable.*

F. H. Westerhoff is coming with simple reformulation of Samuelson model (Westerhoff, 2006). His aim is to obtain lasting endogenous business cycles for more realistic values of parameters  $a$  and  $b$ . As well, he seeks to emphasize the role of expectation formation of player of this game, for business cycles. The past is fitting to our purpose because the formation of expectation by new qualitative states and/or regimes maybe structurally stable. Assuming that expected aggregative output  $E(Y)$  may expressed as a weighted average of all different expectation of players for convenience he divides all their population to two sub-population where in addition, the migration is allow. From our point of view that is good idea because although only in hidden sight points on importancy of network among people. In first sub-population, there are players with an extrapolative and in other one are present players with a regressive expectation rule

$$E(Y_t) = W_t E^e(Y_t) + (1 - W_t) E^r(Y_t). \quad (7)$$

Players presented in extrapolative community either predict a boom (guides to excitation in economic process) or expect a recession (leads down to inhibition), that is formally

$$E^e(Y_t) = Y_{t-1} + c(Y_{t-1} - \bar{Y}), \quad (8)$$

where  $c$  denotes a positive extrapolation parameter and  $\bar{Y}$ , that is  $\bar{Y} = \frac{\bar{G}}{(1-a)}$  is the number is

expressing the long-run equilibrium of aggregative output. In second sub-population the players disposes by regressive expectations to be formalize as

$$E^r(Y_t) = Y_{t-1} + d(\bar{Y} - Y_{t-1}), \quad (9)$$

where the mean-reversion parameter  $0 < d < 1$  take into account the player's expected adjustment speed of the output towards its long-run equilibrium value. Because we want to use for demonstration very suit for that purpose software iDMC beside above model we are using in subsequent part of this essay model of co-author of iDMC M. Lines (Lines, 2006) which is congenial with Westerhoff approach. Her model can serve as very instructive rules for creation models in iDMC and operation with them by the aid of her essay (Lines, 2006) too. In Lines model parameter  $b$  is propensity to consume and parameter  $k$  is the coefficient of acceleration. The population of agents is divided similarly as in Westerhoff to two communities, that is agents in first community has extrapolative believe and the other one believe is based on speed of expectation of the output towards its long run equilibrium value. The migration of agent between two beliefs is possible. The parameter of extrapolative expectation, or trend is  $m1$  and parameter  $m2$  is adjusting speed expectation of the output towards its long-run equilibrium value. From mathematical point of view fixed point  $O$  is stable only if it fulfilled inequality  $b < \frac{1}{k}$ . When two ongoing inequalities fulfilled, that is

$b < \frac{1}{k}$  and  $b < \frac{4k}{(1+k)^2}$  is effective, damped oscillations occur in evolution. To find the

qualitative characters of emerging succession in evolution, that is in virtual iterative process it is suit to use built in manner iDMC for plotting 2D bifurcation portrait. That algorithm can

plot (visualise) Arnolds tongues if such exists in investigated model. If the rate of both analysing parameters is in the inner area of plotted tongue we can intuitively conclude that in that area the model is in the state of its invariant and/or alternative structural stability. The bifurcation 2D portrait of parameter  $k$  against  $b$  is in the snapshot of fig. 6, stratum for enlarging by doing new experiment shows two horizontal lines. The result of chosen layer is in the snapshot of fig. 7.

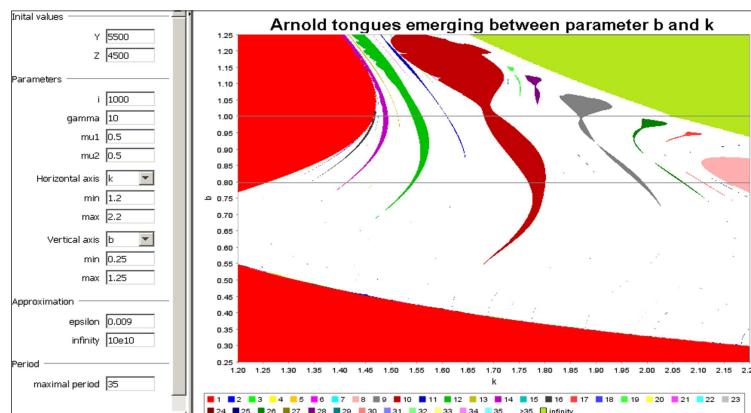


Figure 6

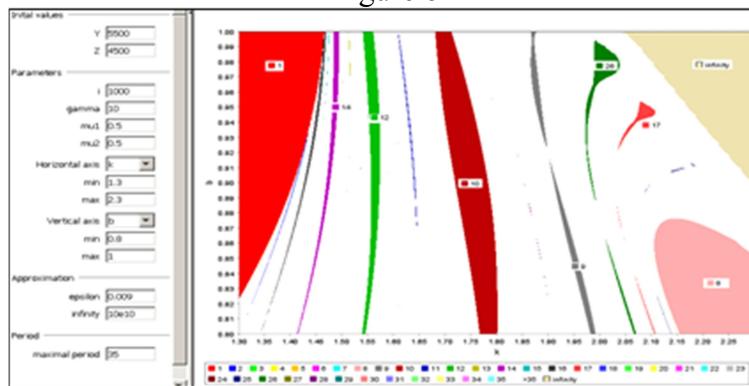


Figure 7

We realised another 2D experiments with bifurcation parametr  $k$  against  $\mu_1$  (scatter form as used in two upper snapshots). In that case temporary business cycles arise due to the interplay of the multiplier and the accelerator however, changes in economic activity either die out or explode (persistent cycles only occur for a nongeneric boundary case).

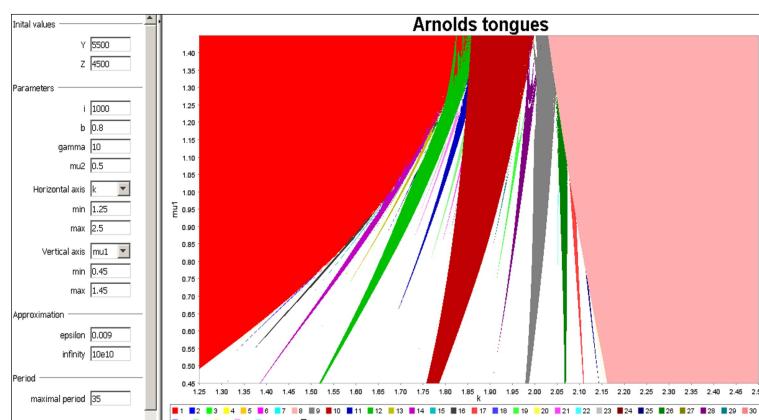


Figure 8

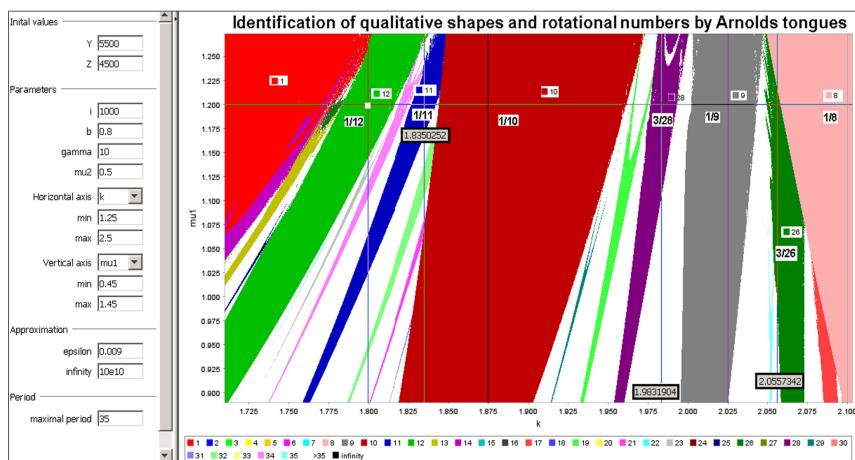


Figure 9

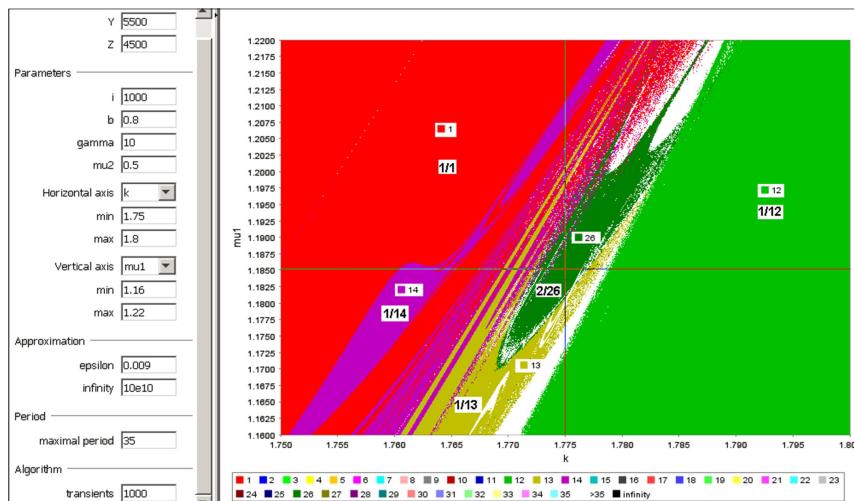


Figure 10

In enlargement of scatter in fig. 9, we used fractions to describe the qualitative character of Arnold's tongues. The numerator describes the number of orbits and the denominator the number of periodic loci's. Figure 11

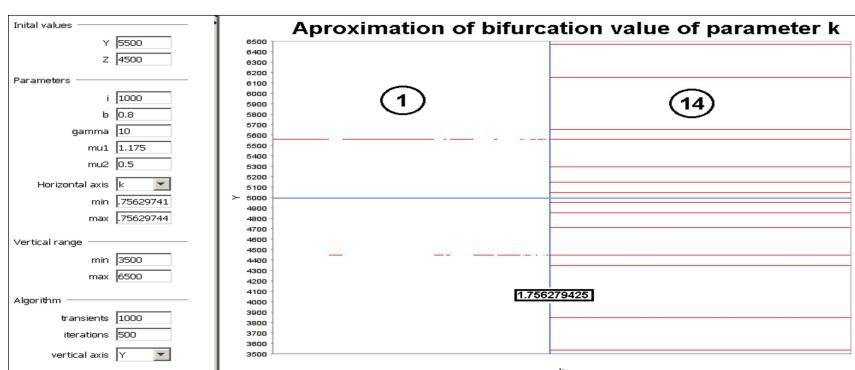


Figure 11

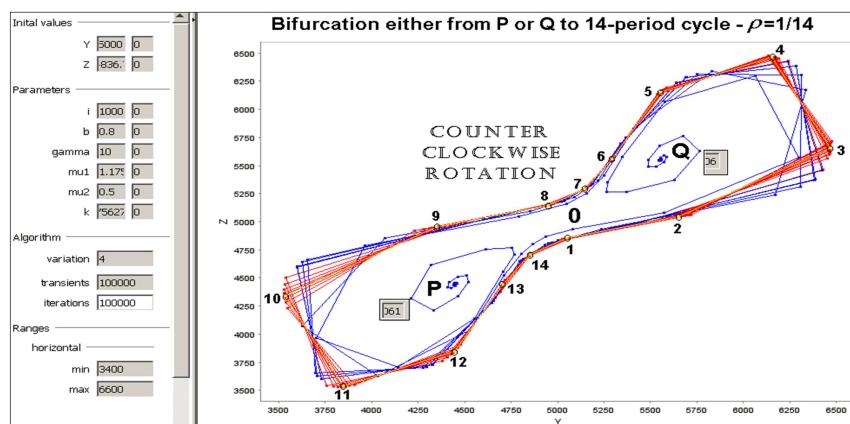


Figure 12

The situation in snapshot of fig. 12 is either *bistable* in *P* and *Q* or *14/period stable* after bifurcation caused by very small perturbation of the parameter *k*. Fixed point *O* is unstable, repellent focus and *P* and *Q* are attractive focuses (to be compared with left and right side of value  $k^* = 1.756279425$  snapshot in fig. 11). Fig. 13 shows plot made by manifold routine.

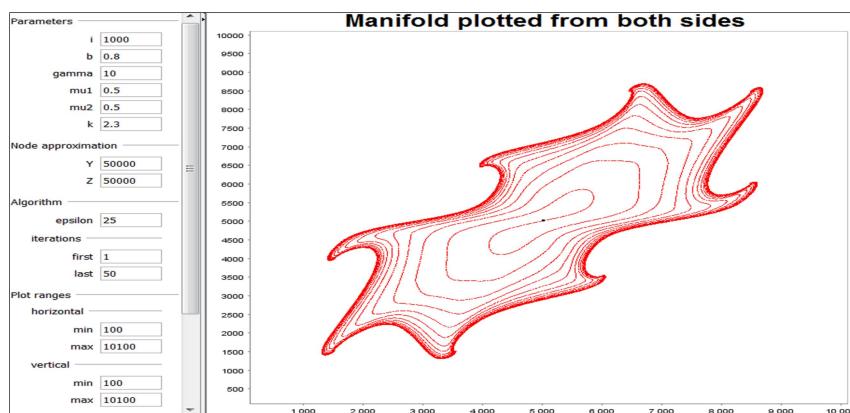


Figure 13

For example, in comparation with upper experiments original M. Lines's bifurcation diagram with two parameters is in the snapshot of fig. 14. She uses for  $k \times \mu_1$  generally for variation in iDMC  $k \in (1, 2.3)$ ,  $\mu_1 \in (0, 1.6)$ ,  $\mu_2 = 0.5$  (fixed in all periods). Specifically in standard constellation of parameters she using subsequent values  $b = 0.8$  (giving  $\Psi = 5000$ ),  $\gamma = 10$ ,  $I_a = 1000$ ,  $Y_0 = 3500$ ,  $Z_0 = 3500$ . She chosen for the algorithm, border of infinity in level  $10e10$ , transient = 5000 with maximal period of = 28 and precision epsilon set = 0.1. The symbol  $\Psi$  introduced for long-run equilibrium output, that is  $\Psi = \frac{1}{1-b} I_a$ . The symbols  $\bar{Y}_1$ ,  $\bar{Y}_2$  stands for alternative (Gemini) results of dual parameter experiment in the are “1“ shown in the snapshot of fig. 14 (may be attractive nodes and/or focuses, and the fixed point *O* is saddle).

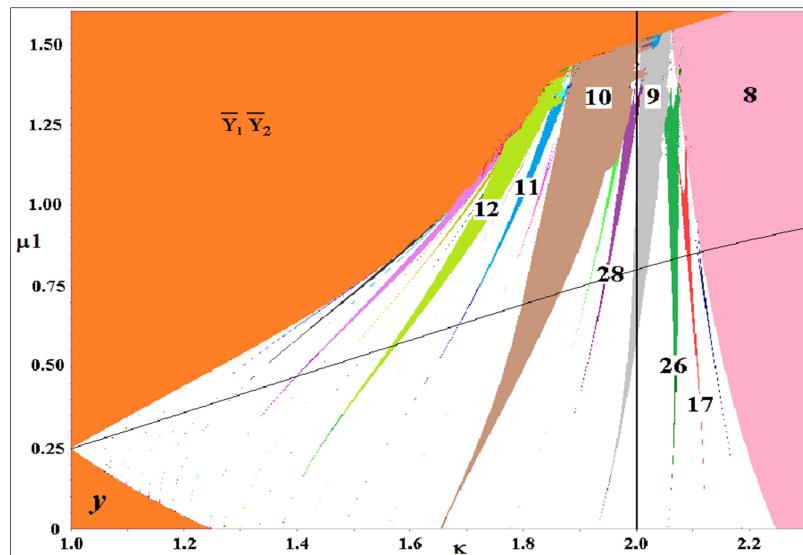


Figure 14

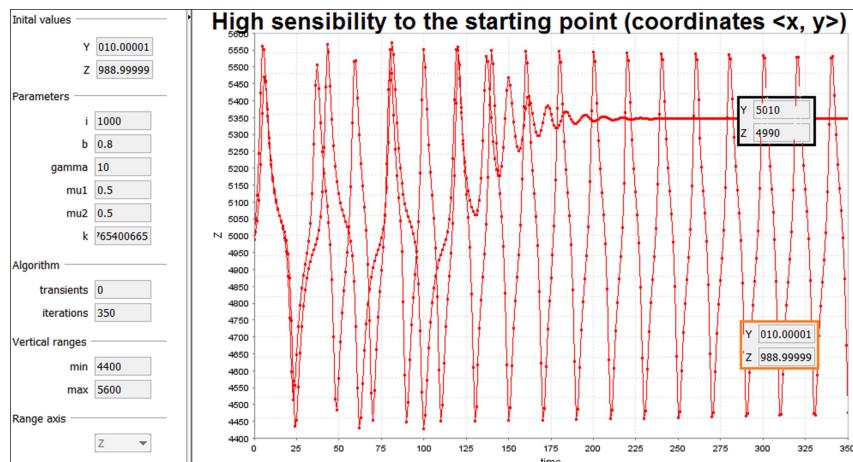


Figure 15

The situation in snapshots fig. 15 -16 show bistable and multistable state (regime).

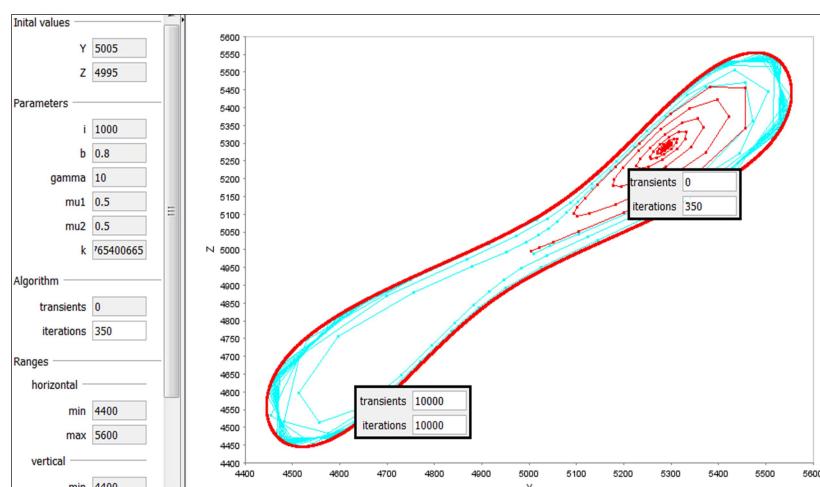


Figure 16

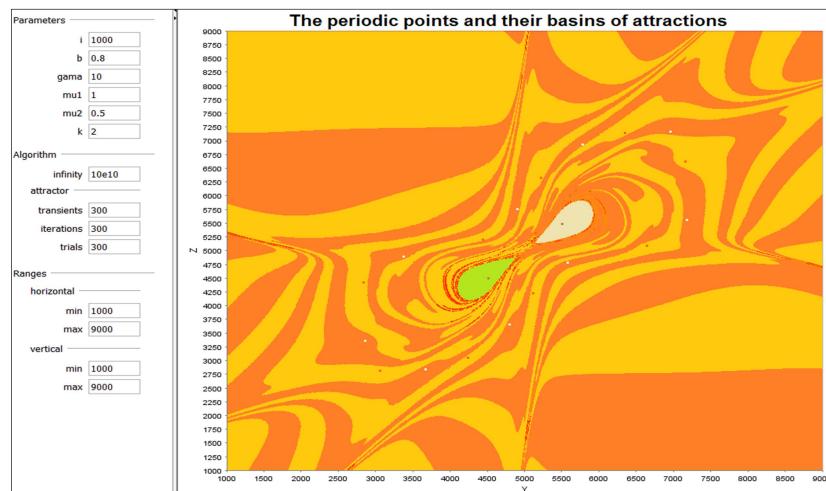


Figure 17

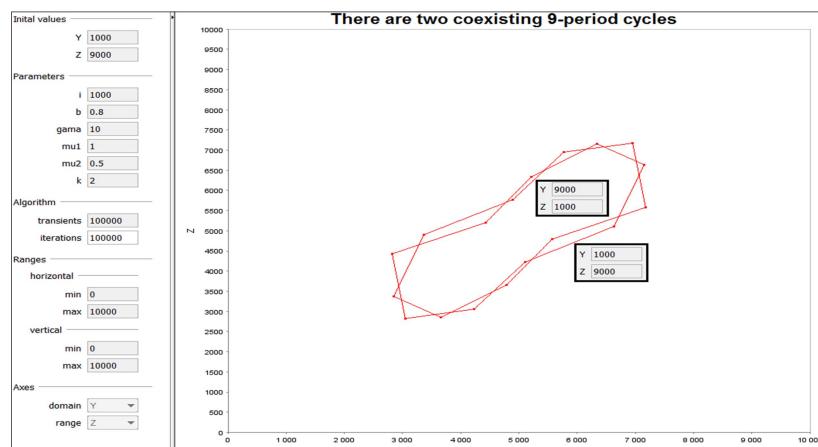


Figure 18

The snapshot of fig. 18 shows the model high-level sensitivity to starting point in variables plane  $< Y, Z >$  of iterative evolution. In fig. 19, there is the bifurcation diagram  $\mu_1$  versus  $Y$ . We refilled it by three vertical lines to show the bifurcation incidents.

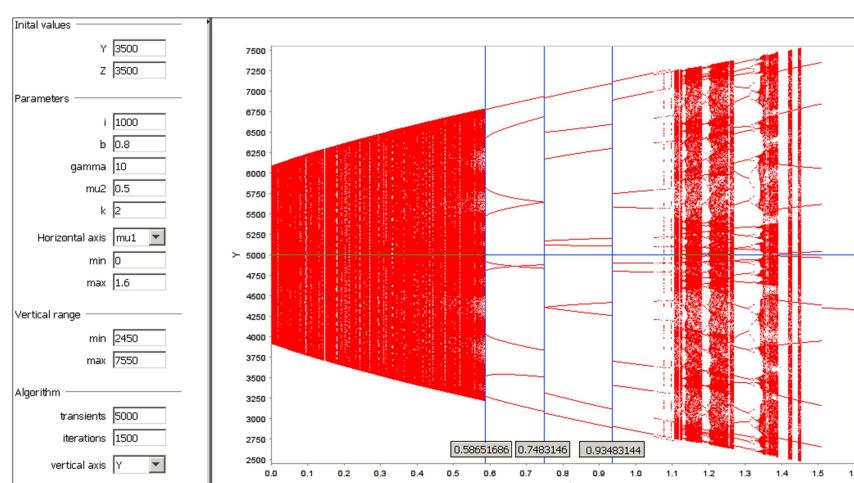


Figure 19

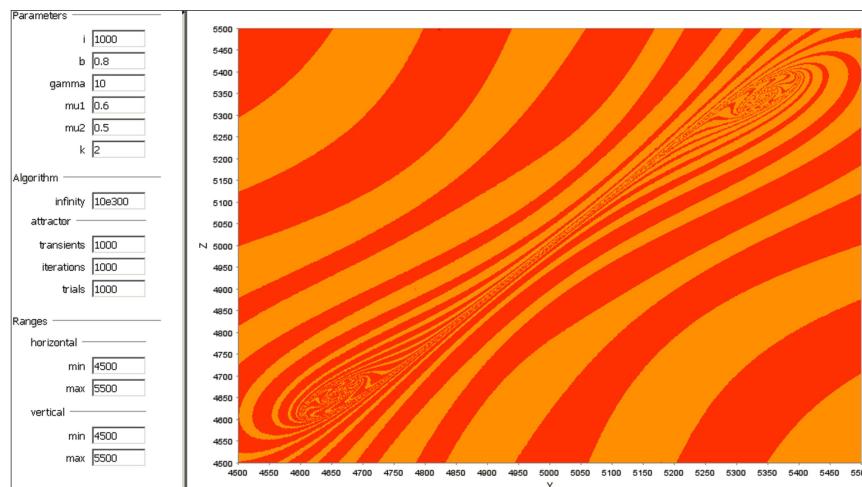


Figure 20

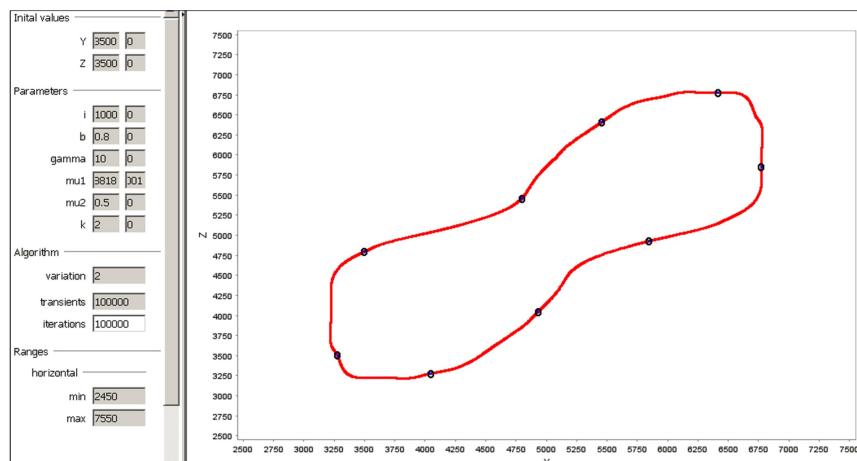


Figure 21

Before bifurcation, that is when  $\mu_1 = 0.58815$  the system rotates as polygon with 9 apexes (nonagon) upon loci's of seemingly closed invariant cycle and after bifurcation that is value of change is  $\Delta\mu_1 = 0.0001$  the system rotates upon nine fixed loci's.

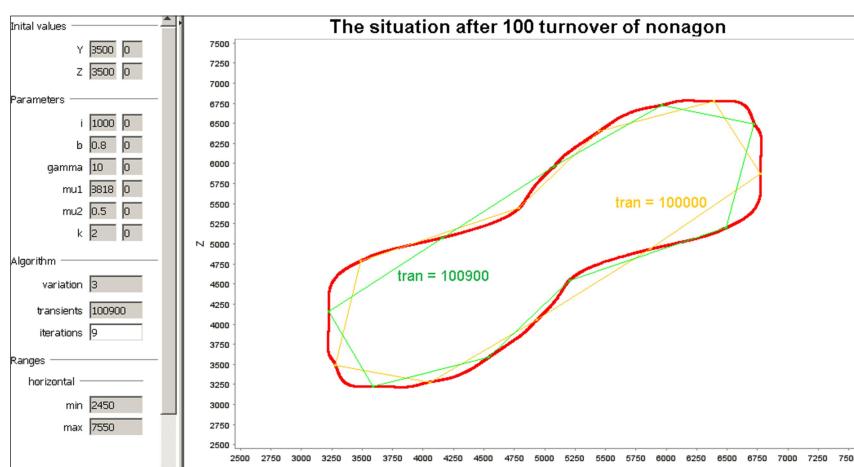


Figure 22

In distinction to state interventions succession adaptively following based on decisions of only single government, in model of Westerhoff-Lines that one is the integral result of pseudo-game of large collection of participants. This is the reason why pseudo-game of chosen type of players carries higher level of mobility to their economic model. As such, it is very good starting platform for advanced imagination on complex phenomena (and cognitive, syntropic entity) as socioeconomic progress is.

### **3. Introducing product and process novelties-innovations as boostser of qualitative movement to higher level of socio-economic progress**

Major mover in socio-economic processes alongside to J. Schumpeter tradition is innovator upcoming with action name creative destruction. The horizontal and vertical diversification of product, technologic process' and services can very successfully twist and roll of the market. Such situation in special cases can lead to entirely new qualitative regime in economies. The very proper example of such case is fundamental discovery of semiconducting in physics and not yet ending concatenation of technological solving based upon that principle up to our days. In general, that concatenation of innovations procreate emergency of information society and in consequences together with other social changes, lead to more sophisticated form of that one which is knowledge based society as novel in historically unpreceded dimension. On the back of that general level of socio-economic motion by in self, yet some of the single novelties and/or innovations appear as monopoly with very high destabilizing potency against existing state in the market. Complexities going along with conversion of that potency to real driving power of changes are for both side of participating player's appropriate show in fact may be strong virtual spectacle of the role of destabilisation via innovation for socio-economic progress for economists. For that purposes we are using simple mental model of monopoly proposed by T. Puu (Puu, 2005, 113-131). In distinct to his general notion of single monopolist we imaging that our player is authentic horizontal innovator coming with entirely new consumption good. Therefore, he/she is in potential status to dispose with a force on the market strong enough to determine prices in a balance with the potential demand. Because his good is exclusively new the expected with innovator buyers is in uncertain state because they can't imagine the utility of thing in the question. For compare of difference in situation, we are using the situation before introducing the first mobiles to the market. In that case the consumers have to knock with it because have experience with wire telephone. Consequently, the innovator must challenge to robust uncertainty resulting in his difficulties with choosing amount of supply and the introductory price. In accordance with current economic reasoning T. Puu uses for stating equation price truncated Taylor series

$$p = A - Bx + Cx^2 - Dx^3, \quad (10)$$

where parameter  $A, B, C$ , and  $D$  are positive constants but unknown,  $p$  is stand for price and  $x$  for quantity demanded. Naturally, the monopolist's task is to find out the values of unknown parameters in iterative adjusting process to get quantity of product to producing and the supply price. It is not easy task to him to look after those quantities, because the potential consumers playing with him complex adaptive game, that process is the *raison d'être* of destabilisation of former market situation. On the other hand, we as analyst's can set on solving process currently used mathematical method and predominantly new mined out methods and tools of CI. In such manner, we can metaphorically emulate the tasks challenged by innovative monopolist. At first what offer mathematic methods. We can also intuitively come to inference that equation (10) must be invertible because to be correspondence need between demanded quantity and its adequate price that is the graph of (10) must be down-

sloping curve. Mathematically the least steep slope is in the point of inflection, because the (10) is cubic function that is its second derivative (equalised to zero) is

$$\frac{d^2 p}{dx^2} = 2C - 6Dx = 0, \quad (11)$$

and the slope is first derivative of (10)

$$\frac{dp}{dx} = -B + 2Cx + 3Dx^2, \quad (12)$$

by substitutin from (11) for  $x = \frac{C}{3D}$ , we get

$$\frac{dp}{dx} = -B + 2Cx + 3Dx^2 < 0, \quad (13)$$

and because the above imposed sign requirement it must be hold the condition

$$C^2 < 3BD. \quad (14)$$

In connection with demand side behaviour uncertainty the innovative monopolist is know his costs and we theoretically can derive the marginal cost equation again as truncated Taylor series, that is

$$MC = E - 2Fx + 3Gx^2, \quad (15)$$

where parameters  $E$ ,  $F$ , and  $G$  are positive constants know to monopolist. Starting with upper information's and from the function of marginal revenue

$$MR = A - 2Bx + Cx^2 - 4Dx^3, \quad (16)$$

we can leaving out some particularities, for saving the place and directly use approach suggested by T. Puu for monopolist's adaptive search, that is based on maximizing of particularly unknown parameters of profit function

$$\Pi(x) = (A - E)x - (B - F)x^2 + (C - G)x^3 - Dx^4. \quad (17)$$

From those preliminary foundations, he put forward retrieval function

$$\frac{\Pi(y) - \Pi(x)}{y - x} = (A - E) - (B - F)(x + y) + (C - G)(x^2 + 2xy + y^2) - d(x^3 + x^2y + xy^2 + y^3), \quad (18)$$

where he used step length  $\kappa$  and tool for accounting difference of values in time delay between two-step dates and for avoiding possible numerical problems in denominator he factored out the quotient from

$$y + \kappa \frac{\Pi(y) - \Pi(x)}{y - x}, \quad (19)$$

and on that base, he formulates 2D topological map  $T$

$$T : \begin{cases} x' = y \\ y' = y + \kappa [a - b(x + y) + c(x^2 + xy + y^2) - d(x^3 + x^2y + xy^2 + y^3)] \end{cases} \quad (20)$$

which we using for experimentation in virtlab created in STELLA and in iDMC. The parameters signed as lower cases created by subtracting former parameters, that is  $a = A - E$ ,  $b = B - F$ ,  $c = C - G$ , and  $d = D$ , using values  $A = 5.6$ ;  $B = 2.7$ ;  $C = 0.62$ ;  $D = 0.05$ ;  $E = 2$ ;  $F = 0.3$  and  $G = 0.02$ . For visual illustration of three curves, we created their mutual coincidence in Excel. In fig. 23 we can see three intersections between marginal revenue and cost curves, which is real roots (and/or radixes) of T. Puu's equation with following numerical values

$$\Pi'(x) = 3.6 - 4.8x + 1.8x^2 - 0.2x^3 = 0. \quad (21)$$

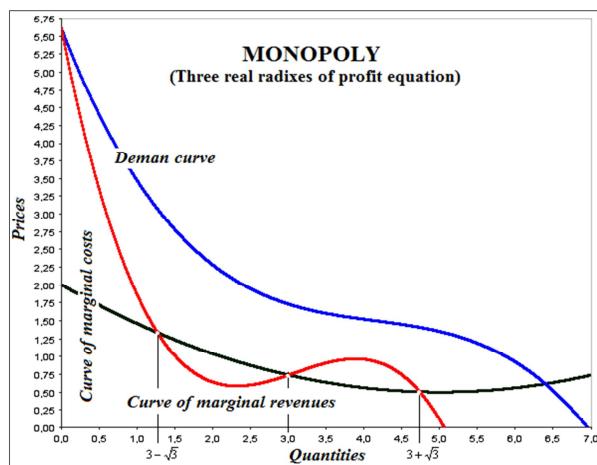


Figure 23

It is favourable first for student because of its nature of leading upon *learning by doing* to constructive reasoning. Nevertheless, exactly for the purposes single followed in this essay it is more convenient to use for demonstrations program routines presented in iDMC.

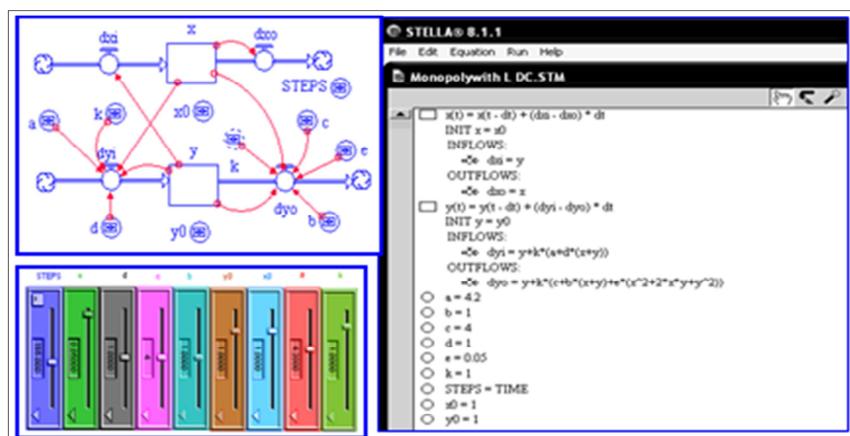


Figure 24

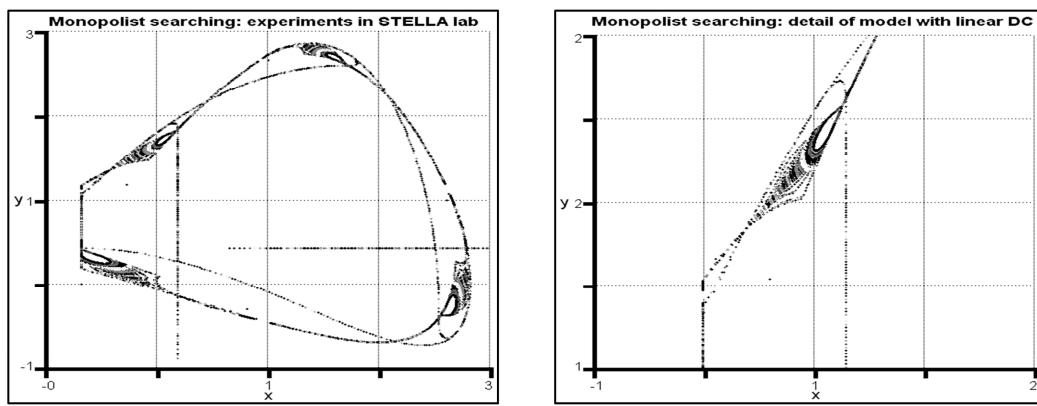


Figure 25

Naturally for understanding complex evolutionary process in virtual economies the best among all approaches is direct dealing with experiments running in PC. For demonstration of iDMC routines potential with investigation of emerging events we presenting some results of

selected simulation runs. We begin that exhibition with time-step trajectory in right snapshot of fig. 26.

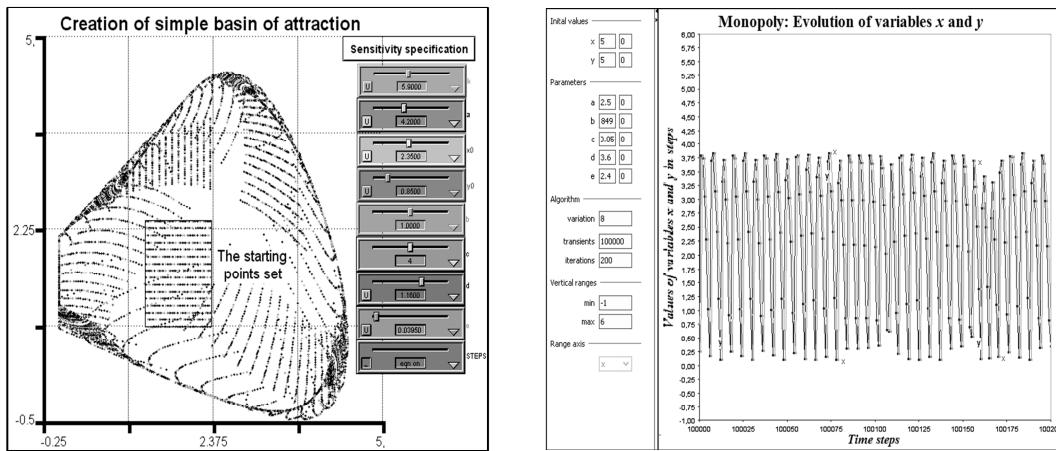


Figure 26

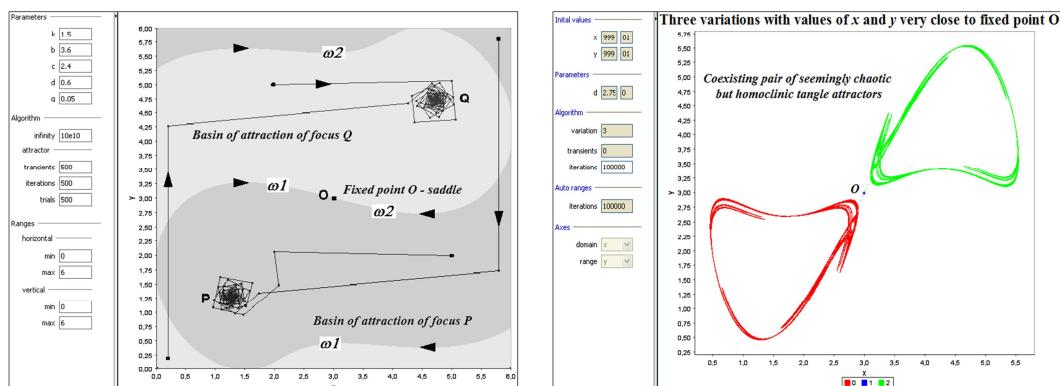


Figure 27

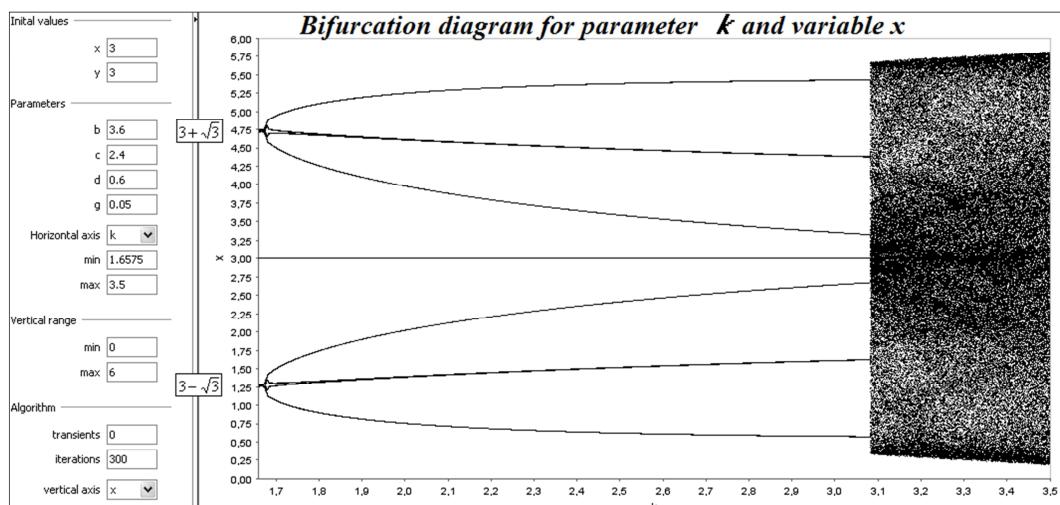


Figure 28

#### 4. Conclusions

In economic science, there are several prejudices, which have in first sight tendency to pretend it as if there are correct scientific presumptions. That is the reason that sometimes it is very difficult to reveal their existences. There is however worse reality based on stigmergy and/or herding tendency in people behaviour answering such bad signals. That type of behavior is contributing to fast spread of such false imaginations.

We think that scientist's imaginations with reality were obliged to take into account and using several approaches enclosure uncertainty when they can't understand the thing directly as deterministic phenomena. On the other hand fact is that the majority of contemporary scientists immediately and directly take into account uncertainty as a state of things and not a merely provisional failure of cognitive nature without intention to look after more direct solving. At present we have no base for correct conclusion if the objective reality is deterministic or not. Maybe every consequence has its own reason – we don't know. The science is not called up to producing Sibylline Prophecies about such matter but for searching objective facts. The existence of enigma certainty-uncertainty in objective reality isn't correctly approved but the theory and methods of probability theory that's just it are useful. It is absolutely correct conclusion that in long period we (that is live today) becoming dead. Actually, J. M. Keynes is right in that fact but his static theory of effective demand as mover of economy not, it is at least second best potency. Consumer can reject new products and services but such rejection cannot destabilise the situation in the market just opposite is true. In reverse case when they adopt newness, they help to create new structural stability of economy in the role of secondary force of movement. Working with subjectively created mental models we are in wholly different and more convenient positions. Even though the acquired know-how with ICT and CI products and service in cultivation of economic imagination and/or reasoning in complex processes like several instabilities we have in mind exceedingly the quality of economic reasoning not immediately the objective economic reality. In the process of transition, our theoretical results on objective existing phenomena to interpret it everybody and great of economics mages too, have to be very careful. Theory is only the product of human genius and objective reality of spontaneous evolution in the case of human activities with aspiration to realise famous Carl Marx's eleventh thesis on Ludwig Feuerbach: "Philosophers have hitherto only interpreted the world in various ways; the point is to change it". It is luckily for next generations, the objective reality resting forever be very independent from human megalomania and behaving by own laws. The right is another great genius of humankind Johann Wolfgang Goethe: "All theory, dear friend, is gray, but the golden tree of life springs ever green." Moreover, that golden tree started evolving because of greatest fluctuation in Universe after Big Bang, astrophysicist maintains.

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